

## OPTIMAL COSTS OF QUALITY

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**It can be noticed that our suppliers monitor the quality of their process results (semi-product, product, documentation, service) mainly through a percentage of malfunctions or a decrease in the index of accuracy and precision of the process. However, in the United States they claim that their users are not interested in it and that defect rates are not quality losses, but costs of suppliers. In addition, in the United States they claim that *PPC* (capability index) and *PPP* (accuracy index) are difficult to understand because "what really is a quality improvement when the *PPC* changes from 0.9 to 1.2". Therefore, the quality of their process results in the average cost of quality which includes all the costs of quality at all stages of the production process (market research, planning, design, production, control, packaging, sales, servicing and maintenance) and the work with one worker who performs five jobs: production, quality control, adjustment, release to accept correct, repaired or regrade and scrap incorrect process results, with preventive maintenance, instead of our four jobs (production, adjustment, quality inspector and maintenance worker. In this release, we briefly discussed possible ways of calculating optimal quality costs, which can also enable the organization of jobs in a new, American way.**

**Keywords:** Cost of quality, Optimal cost of quality.

### INTRODUCTION

Today, in the contemporary industrial production of the product (semi-product, product, documentation, service), a different working procedure is applied than in our country. We still have four jobs in the industry: a production worker, an adjuster, an inspector and a maintenance manager for four types of jobs. A workman on the equipment (apparatus, device, machine) produces the results of the process, the adjuster adjusts the production process, the inspector performs the quality control of the process results, and the maintenance worker performs preventive maintenance of the equipment (Taguchi, 1981).

In America, all four jobs only have one workplace process auditor (checker) who performs process recoveries with five jobs: production, inspection, adjustment, release, and maintenance. Manufacturing uses the technology of converting the raw material into the result of the process. The

inspection includes detection of malfunctions and defects in the quality of process results. Adjusting allows correcting incorrect processes.

The release includes downloading the correct process results and delaying process results for correction and repairing, regrade or scrap. The final maintenance work includes preventive maintenance of the working equipment. Of course, in America, automatic production systems are introduced on a daily basis due to higher quality and replacement of workers. In this release, certain problems of process verification are considered: the cost of quality, optimal costs of planning process tolerances, optimum cost of process quality and optimal costs of adjusting the accuracy of the process (Popović, 2018).

### METHODOLOGY

The conducted analysis of the general costs of quality and its own acquired practice in the industry has shown that we still do not have

detailed considerations of concrete quality costs. Therefore, the methodology of statistical cost analysis, which arises in the planning of tolerances, quality control, and process adjustment, has been applied here. The total cost of quality has been defined and also optimal quality costs are determined, with minimum amounts. In addition to the application of classical standardized Normal Distribution, Uniform Statistical Distribution was also used.

Optimal costs of quality content the minimal cost of quality in process verification, which are defined according to a certain optimum. Defining the cost of quality requires a statistical consideration of the cost of quality that arose during the realization of the value of the observed product size in relation to the tolerance and its limits. The product has in its representative sample its known values  $X(x_1, x_2, \dots, x_n)$  and statistical parameters: sample size ( $n$ ), average ( $\bar{x}$ ), standard deviation ( $s$ ) and variance ( $s^2$ ) deviation of the measured sample values (Taguchi et al., 2005).

These empirical values of the sample belong to a set of values ( $N$ ), with unknown statistical parameters: the size ( $N$ ), the average ( $\mu$ ), the standard deviation ( $\sigma$ ), and the variance ( $\sigma^2$ ) of the deviation. However, although the values of the set are unknown, they can be estimated according to certain theoretical statistical distributions and their laws  $f(x)$  and the distribution functions  $F(x)$ .

The most useful is the normal distribution of  $N(\mu, \sigma)$  which can go into the standardized Normal distribution  $N(0, 1)$ , with the transformation  $z = (x - \mu)/\sigma$ .

The standardized distribution has the law of probability  $f(x)$  in the form of a bell curve according to the scheme in Figure 1 with values of variable size  $Z(z_1, z_2, \dots, z_n)$  within the space ( $-3 \leq z < +3$ ), between the lower ( $z = -3$ ) and upper ( $z = 3$ ) limits of tolerance.

Therefore, if the values of a particular product are observed, then the distribution function  $F(x)$  defines the high probability (0,9973) of the occurrence of a variable within the tolerance range ( $\pm T$ ) and the low probability (0,0027) of the appearance of the variable product outside this area (Popović, 2016).

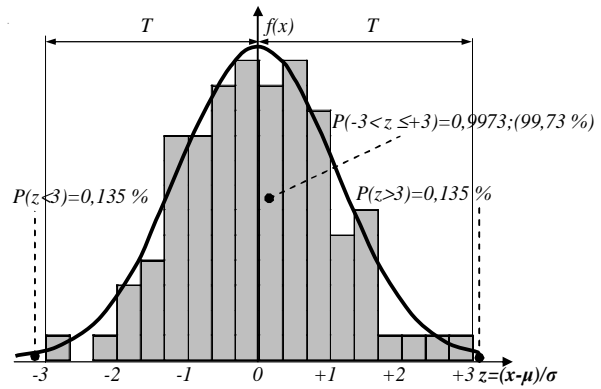


Figure 1: Sample Histogram and Curves of Standardized Normal Distribution Laws

The average cost of quality  $c(x)$  depends on the position of the value of the functional characteristics of the product quality ( $x$ ) in relation to the tolerances ( $\pm T$ ) and the average ( $\mu$ ), according to the cost function  $f(x)$  in the form of a concave curve according to Figure 2. Tolerance the prescribed area within which the realized value of the size of the results of the process should be found, the same tolerances have the same value ( $\pm T$ ) and unequal different values ( $T_1, T_2$ ).

Tolerance limits are the extreme values of the tolerance area, e.g. the limits of equal tolerances ( $-T$ ) and ( $+T$ ). These costs start at the beginning ( $x = \mu - T$ ) from the maximum error value ( $C_D$ ) to the average value ( $x = \mu$ ) and then again grow to the value ( $C_D$ ). Obviously, at the center point ( $x = \mu$ ) there are optimal (minimum) average cost of quality  $c(x) = 0$ , which should be defined (Popović & Ivanović, 2011).

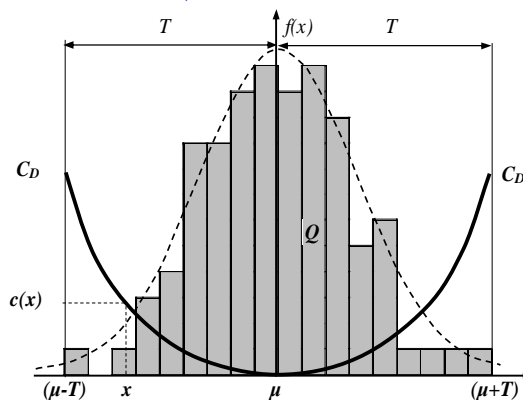


Figure 2: Defects of laws Normal distribution and quality cost functions.

The function of the average cost of quality  $c(x)$  depends on the values ( $x$ ) and the average ( $\mu$ ) according to the formula:

$$c(x) = f(x, \mu), \quad (1)$$

so if the deviation from the average is negative ( $x < \mu$ ) or positive ( $x > \mu$ ) then the cost of the defect is calculated according to the formula:

$$c(x) = c(+\mu + x - \mu). \quad (2)$$

It is obvious that this continuous function has all derivatives and it can be developed into Taylor's order:

$$c(x) = c(\mu) + \frac{c'(\mu)}{1!}(x - \mu) + \frac{c''(\mu)}{2!}(x - \mu)^2 + \dots \quad (3)$$

At the point of the average value ( $x = \mu$ ) where the costs are equal to zero  $c(x) = 0$ , the first two members of the function are equal to zero, and other members of the function with outputs higher than the second degree can be ignored, so that optimum quality costs :

$$c(x) = \frac{c''(\mu)}{2!}(x - \mu)^2, \quad (4)$$

where the amount in the fraction is a certain proportional cost constant:

$$K = \frac{c''(\mu)}{2!}, \quad (5)$$

so when the size ( $x$ ) deviates from the value of the average size ( $\mu$ ) for the tolerance ( $T$ ), then the cost of eliminating the defect ( $C_D$ ) of the product is generated, with a constant:

$$K = \frac{C_D}{T^2}, \quad (6)$$

so that the optimum cost of the quality function with the variance variance ( $\sigma^2$ ) is obtained:

$$c(x) = \frac{C_D}{T^2}(x - \mu)^2 = \frac{C_D}{T^2}\sigma^2. \quad (7)$$

## COSTS OF PLANNING PROCESS

Optimal costs of planning process tolerances include consideration of one or more tolerances, equal or uneven tolerances, tolerance of part or set of parts, nominal, minimum and maximum tolerances. Nominal tolerances (The-Nominal-The-Better, N-type) are most commonly occurring, e.g. in the case of product sizes or gaps, when defective products may appear outside the tolerance ( $\pm T$ ), when their values are below the minimum threshold ( $-T$ ) or above the maximum upper limit ( $+T$ ) of tolerance.

Minimal tolerances (The-Larger-The-Better, L type) are generated, for example, when tolerating the desired higher reliability or resistance of the construction of the product, when their values may be below the lower limit ( $-T$ ) of tolerance.

Maximum tolerances (The-Smaller-The-Better, S type) are most often calculated, for example, when tolerating undesirable process of wearing parts or deterioration of product performance, when their values can be above the upper limit ( $+T$ ) tolerance (Popović & Bošković, 2011; Popović & Klarin, 2007; Popović & Klarin, 2003).

In planning tolerances with optimal quality costs, one should distinguish the planned ( $\mu_0, T_0$ ) and the required ( $\mu, T$ ) the size and size tolerances ( $x$ ), as well as the planned ( $c_{D0}$ ) and the required ( $c_D$ ) costs of the malfunction, in order to get minimum cost of quality, for which it is necessary to provide the following condition, using the formula (7) obtained:

$$\frac{c_D}{T^2}(x - \mu)^2 \leq \frac{c_{D0}}{T_0^2}(x - \mu)^2, \quad (8)$$

from where the optimal optimum value of tolerance is obtained:

$$T = \pm \sqrt{T_0^2 \frac{c_D}{c_{D0}}} = \pm T_0 \sqrt{\frac{c_D}{c_{D0}}}. \quad (9)$$

## Results of the planning costs

The application of the adopted methodology gave concrete results of the costs of quality in the case of planning tolerances in the design of one simple product. For example, consider planning the necessary tolerance of the home Water heater with

the functional characteristic of the temperature temperature  $\mu_0 = 80$  [°C] with the planned tolerance  $T_0 = \pm 15$  [°C]. Due to possible temperature deviation due to atmospheric effects, the temperature thermostat should sometimes be replaced with the cost of  $c_D = 1$  \$, since for each defective water heater, the cost of  $c_{D0} = 200$  \$ .

Using the obtained formula (6) calculates the required optimum value of the quality, with the minimum cost of quality, it is:

$$T = \pm T_0 \sqrt{\frac{c_D}{c_{D0}}} T = \pm 15 \sqrt{\frac{1}{200}} = \pm 1.06 [^{\circ}C].$$

The results obtained are shown in Figure 3 according to which the manufacturer must use the planned temperature value ( $\mu_0 = 80$  [°C]) with the required optimum tolerance ( $T = 1.06$  [°C]) in order to ensure the minimal quality of production costs.

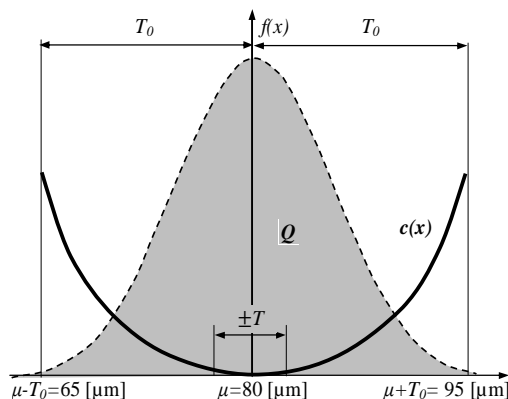


Figure 3: Scheduled ( $\pm T_0$ ) and required tolerances ( $\pm T$ )

### COSTS OF THE QUALITY CONTROL

Quality costs require consideration of: quality costs without (0%) verification of product quality, product quality control costs through sample ( $\approx 10\%$ ), and quality control costs of all (100%) products. Costs of quality without (0%) checking of the product, arise when products are considered to have "probably good" quality, and it is not necessary to control the quality characteristics.

These quality costs can be calculated according to the Standardized Normal Distribution or the Equivalent Distribution when the defective products are located outside the area of Tolerance ( $\pm T$ ) (Popović & Klarin, 2002; Popović &

Todorović, 2000; Popović, 1993).

Standardized  $N(0, 1)$  Normal distribution has the following: random variable size, standard deviation, probability law, and statistical distribution function:

$$z = \frac{x - \mu}{\sigma}, \quad -\infty < x < +\infty;$$

$$\sigma = \frac{2T}{6}; \quad f(x) = \frac{1}{\sqrt{2\pi}\sigma^2} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx. \quad (10)$$

According to the Normal Distribution Scheme in Figure 4, outside the tolerance region ( $\pm T$ ) there are defective products, with a probability of 0.0027 or a proportion 0.27% (Popović & Klarin, 2005; Popović et al., 2008).

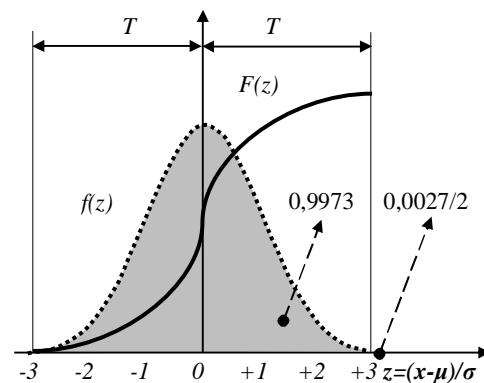


Figure 4: Probability law and functions of standardized Normal distribution

By applying formula (7) the optimal cost of quality characteristics with Normal distribution is:

$$c_N(x) = \frac{c_D}{T^2} \sigma^2 = \frac{c_D}{T^2} \left( \frac{2T}{6} \right)^2. \quad (11)$$

The Uniform continuous distribution has the following: random variable size, standard deviation, probability law, and statistical distribution function:

$$x, \quad -\infty < a < b < +\infty; \quad \sigma = \frac{2T}{\sqrt{12}}. \quad (12)$$

According to the scheme of the plane distribution

in Figure 5, outside the tolerance region ( $\pm T$ ) there are defective products, with a probability of 0.00 or a proportion of 0.00%. Using the formula (7), we obtain the formula of optimal cost of quality control of the quality of the product with the Uniform distribution:

$$c_R(x) = \frac{c_D}{T^2} \sigma^2 = \frac{c_D}{T^2} \frac{2T}{\sqrt{12}}. \quad (13)$$

For example, consider possible costs without checking the quality  $N = 1,000$  products with tolerances  $T = \pm 5 \mu\text{m}$ , whereby each defective product can be refined or rejected with the cost of  $c_D = 7 \$$ , as it is considered to have "probably good" quality.

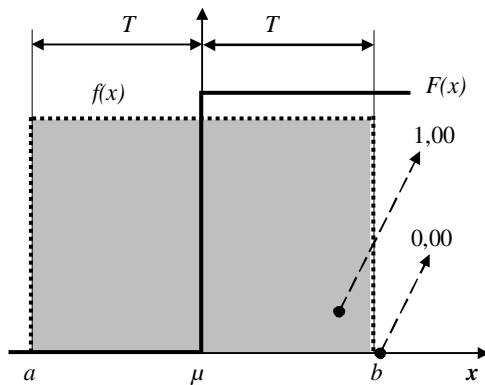


Figure 5: Law on probability and functions of Uniform distribution

Using the obtained formulas (5) and (7), the following average and total cost of the quality were calculated, without checking the quality, according to the Normal Distribution and the Uniform Distribution:

$$c_N(x) = \frac{c_D}{T^2} \left(\frac{2T}{6}\right)^2 = \frac{7}{5^2} \left(\frac{2 \cdot 5}{6}\right)^2 = 0.777 \$ / kom$$

$$C_N(x) = N \cdot c_N(x) = 1,000 \cdot 0.777 = 777 \$.$$

$$c_R(x) = \frac{c_D}{T^2} \frac{2T}{\sqrt{12}} = \frac{7}{5^2} \frac{2 \cdot 5}{\sqrt{12}} = 0.808 \$ / kom;$$

$$C_R(x) = N \cdot c_R(x) = 1,000 \cdot 0.808 = 808 \$.$$

Costs for quality control over samples ( $\approx 10\%$ ) of the product are generated when the results of the process are considered to have "acceptable" quality, and it is sufficient if only the control of a smaller sample ( $\approx 10\%$ ) of the product is applied.

These quality control costs through product samples can be calculated according to the

standardized Normal distribution, because within the tolerance areas there is a variable product with a high probability ( $P = 0.9973$  or a proportion of 99.73%). Average cost of quality includes the sum of: average costs of controlling the quality characteristics ( $c_C$ ) of the product, the average cost of elimination of malfunction ( $c_D$ ), the probability of ( $q$ ) defective products and the average optimum cost of the quality function (Taguchi, 1981):

$$c(x) = c_C + c_D \cdot q + \frac{c_D}{T^2} \sigma^2. \quad (14)$$

### Results of the quality costs

The application of the adopted methodology gave concrete results of the cost of quality in cases when there is no quality review, in the control of samples and in the control of all samples of the product. For example, we consider the cost of controlling the quality of  $N = 1,000$  products through a sample of the product, with tolerances  $T = \pm 5 \mu\text{m}$ , costs of possible finishing or rejection = 7 \$ and control costs  $c_C = 0.03 \$$ , since products are considered to have "acceptable" quality. Using the obtained formula (10) standardized Normal distribution has a function with probability:

$$F(x) = \frac{1}{2\pi} \int_{\mu-5}^{\mu+5} e^{-(1/2)(6/10)^2(x-\mu)^2} dx = 0.9973,$$

from where the variance of variance can be obtained:

$$\begin{aligned} \sigma^2 &= \frac{1}{0.9973} \int_{\mu-5}^{\mu+5} \frac{1}{\sqrt{2\pi}} \frac{6}{10} (x-\mu)^2 e^{-(1/2)(6/10)^2(x-\mu)^2} dx = \\ &= \left(\frac{10}{6}\right)^2 0.986^2 = 2.70 [\mu\text{m}^2]. \end{aligned}$$

Using the formula (13), optimum average costs and total costs are obtained, quality control through a sample for the probability of defective  $1-0.9 = 0.1$ :

$$\begin{aligned} c(x) &= c_C + c_D \cdot p + \frac{c_D}{T^2} \sigma^2 = \\ &= 0.03 + 7 \cdot 0.0027 + \frac{7}{5^2} \cdot 2.70 = 0.804 \$ / kom, \\ C(x) &= N \cdot (1-0.9) \cdot c(x) = 1,000 \cdot 0.1 \cdot 0.804 = 80 \$ . \end{aligned}$$

Costs of quality control of all (100%) products arise when they are considered to have "probably

poor" quality, so it is necessary to control the quality characteristics of all products.

These quality costs can be calculated according to standardized Normal Statistical Distribution, where defective products are found outside the area of Tolerance ( $\pm T$ ).

For example, we consider the costs of controlling the quality of all  $N = 1.000$  products, with tolerances  $T = \pm 5$  [ $\mu\text{m}$ ], cost of possible finishing or rejection  $c_D = 7$  \$ and control costs  $c_C = 0.03$  \$, since products are considered to be "likely bad" quality.

Using the formula (13), the optimum average and total cost of controlling all (100%) products are obtained, with the probability of defective  $p = 1 - 0.9973 = 0.0027$ :

$$\begin{aligned} c(x) &= c_C + c_D \cdot p + \frac{c_D}{T^2} \sigma^2 = \\ &= 0.03 + 7 \cdot 0.0027 + \frac{7}{5^2} \cdot 2.70 = 0.804 \text{ $ / kom,} \\ C(x) &= N \cdot c(x) = 1,000 \cdot 0.804 = 804 \text{ $.} \end{aligned}$$

Based on the results obtained, now all three types of product quality control and control can be compared.

The highest costs of controlling the quality of all products (804 \$) and the lowest cost controlling through a sample product (80 \$). In addition, control over a sample of products has an even greater advantage over costs without checking (777 \$ ÷ 808 \$), as it does not require a significant time controlling.

## COSTS OF ADJUSTING

Optimal costs of adjusting the accuracy of the process include considerations of process parameter settings, that is, functional characteristics of the quality of process results, which are controlled and compared with the prescribed tolerances, in order to achieve the quality of the product. Optimal process setup costs include minimal cost of quality in the process parameter setting.

Subsequent adjustment is done after trial production, when the correction of the incorrect process is achieved, so that it is in accordance with

the prescribed size, ie it becomes the correct process. The current adjustment is done during the process realization, using automatic adjustment systems. The process of adjusting the accuracy of the process requires the shift of certain boundaries, in order to change the size of the average ( $\bar{x}$ ) of the value of the product size  $X(x_1, x_2, \dots, x_n)$  in relation to the process tolerances ( $\pm T$ ).

The accuracy settings of the process require the calculation of: optimum adjustment costs, optimal adjustment intervals, and optimum setup sizes. The size of the accuracy of the process according to the scheme in Figure 6 is:

$$\Delta = x - \mu - (\bar{x} - \mu) = (\bar{x} - \mu). \quad (15)$$

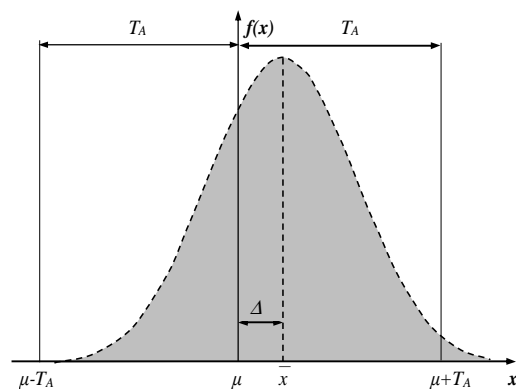


Figure 6: Size of process accuracy adjustment

The expectation of variance of the deviation of the value ( $x$ ) of the process from the average ( $\mu$ ) is:

$$\begin{aligned} E(\sigma^2) &= E(x - \mu)^2 = \\ &= E(x) + [E(x) - \mu]^2, \end{aligned} \quad (16)$$

so the setting value (Taguchi & Chowdhury, 2005; Taguchi, 1978; Taguchi, 1978):

$$\Delta = (\bar{x} - \mu) = \frac{1}{n} \sum_{i=1}^n (x_i - \mu). \quad (17)$$

with variation of deviation and with degree of freedom ( $n-1$ ). due to statistical estimation of average values ( $\mu$ ):

$$\sigma^2 = \frac{1}{n-1} \left[ \sum_{i=1}^n x_i^2 - \frac{\left( \sum_{i=1}^n x_i \right)^2}{n} \right]. \quad (18)$$

**Results of the adjusting costs**

The application of the adopted methodology gave concrete results of the cost of quality in cases functional quality characteristics with tolerances of diameter, and the heating temperature parameter when Hot pressing the product. For example, consider the optimum cost of adjusting the accuracy of the process, functional quality characteristics with tolerances of diameter  $T = \pm 0.6 \mu\text{m}$  of the product, which is processed daily in an amount of 8.000 pcs, where each defective product

$$\sigma^2 = \frac{1}{n-1} \left[ \sum_{i=1}^n x_{li}^2 - \frac{\left( \sum_{i=1}^n x_{li} \right)^2}{n} \right] = \frac{1}{19} \left[ 0.3^2 + 0.0^2 + \dots + 0.2^2 - \frac{2^2}{20} \right] = 0.0684 [\mu\text{m}^2],$$

$$c(x) = \frac{c_D}{T^2} \sigma^2 = \frac{0.30}{0.6^2} 0.0684 = 0.0570 \text{ \$ / kom.}$$

The optimum cost of adjusting process accuracy has adjustment tolerance ( $T_A$ ), which depends on the relationship between the correct and defective products. If the process adjustment thresholds are

$$\begin{aligned} \sigma^2 &= \frac{b-a}{12} = \frac{[(\mu+T_A) - (\mu-T_A)]^2}{12} = \\ &= \frac{(\mu+T_A)^2 - 2(\mu+T_A)(\mu-T_A) + (\mu-T_A)^2}{12} = \\ &= \frac{4T_A^2}{12} = \frac{T_A^2}{3}, \end{aligned} \tag{19}$$

so the costs due to the deviation of the product size from the tolerance area ( $\pm T_A$ ) according to formula (7) will be:

$$c(x) = \frac{c_D}{T^2} \sigma^2 = \frac{c_D}{T^2} \frac{T_A^2}{3}. \tag{20}$$

In the process of checking and controlling the quality of the result of the process, different outcomes arise, with the numbers of the correct and defective products according to Table 1, so that after the  $n$ -th check of the process, the total number of defective products is obtained  $[n(n+1)]/2$  and the next average number of defective products:

$$\frac{n(n+1)}{2} = \frac{n+1}{2}, \tag{21}$$

can be refined or rejected with average cost of defective  $c_D = 30 \text{ \$}$ . The following deviations of the characteristics of the diameter quality were measured: 0.3; 0.0; -0.1; 0.0; 0.3; 0.2; 0.1; -0.2; 0.6; 0.4; -0.2; 0.1; 0.0; -0.4; 0.5; 0.4; -0.2; 0.0; 0.0; 0.2  $\mu\text{m}$ .

Using the formula (18), the required deviation variance value is obtained and using the formula (7) gives the optimum cost of adjusting the accuracy of the process:

set at the ends of the process tolerance values ( $\pm T_A$ ), then the value of the variance of the uniform distribution of the correct products is obtained in accordance with Figure 5:

*Table 1: Process verification with correct (○) and defective (●) products*

Consecutive checks	Cases (j)	A string of checks							Total defective	
		i						i+1		
Possible outcomes	1	○	○	...	...	...	○	○	●	$\frac{n(n+1)}{2}$
	2	○	○	...	...	...	○	●	●	
	⋮	○	○	...	...	...	●	●	●	
	⋮	...	...	...	...	...	...	...	...	
	n-1	○	●	...	...	...	●	●	●	
	n	●	●	...	...	...	●	●	●	
Defective products		1	2	...	...	...	n-2	n-1	n	

After verification, the deviation of the product size ( $x$ ) from the tolerance area ( $\pm T$ ) can be immediately detected, so the variance variance is proportional to the number of defective products, with the process check interval ( $n$ ), the planned interval of adjustment ( $n_A$ ) accuracy and the leakage interval of the product process ( $n_L$ ):

$$\sigma^2 = \left( \frac{n+1}{2} + n_L \right) \frac{T_A^2}{n_A}. \quad (22)$$

All average quality costs according to Formula (20) must now include a sum of: total cost of control ( $C_C$ ) with process check interval ( $n$ ), total cost of adjustment ( $C_A$ ) with planned interval of adjustment ( $n_{A0}$ ), total cost of failure ( $C_D$ ) tolerances ( $\pm T$ ), costs according to the quality cost function and possible cost of control error ( $\sigma_M^2$ ).

The admission of variance (22) into the developed formula (7) gives the cost of quality before adjusting:

$$c(x) = \frac{C_C}{n} + \frac{C_A}{n_A} + \frac{C_D}{T^2} = \left[ \frac{T_A^2}{3} + \left( \frac{n+1}{2} + n_L \right) \frac{T_A^2}{n_A} + \sigma_M^2 \right]. \quad (23)$$

The optimum adjustment interval ( $n_A$ ) and the optimum tolerance of the settings ( $T_A$ ) can be calculated by the derivation of the obtained formula (23).

Here we need to distinguish the planned ( $n_{A0}$ ,  $T_{A0}$ ) and the required ( $n_A$ ,  $T_A$ ) the size of the average and the tolerance of the settings, and similar to the formula (9), the value of the required adjustment interval is obtained:

$$n_A = n_{A0} \frac{T_A^2}{T_{A0}^2}. \quad (24)$$

By incorporating this formula (24) in formula (23), the required quality costs are obtained after adjusting:

$$c(x) = \frac{C_C}{n} + \frac{C_A}{n_{A0} \frac{T_A^2}{T_{A0}^2}} + \frac{C_D}{T^2} \left[ \frac{T_A^2}{3} + \left( \frac{n+1}{2} + n_L \right) \frac{T_A^2}{n_{A0} \frac{T_A^2}{T_{A0}^2}} + \sigma_M^2 \right] = \frac{C_C}{n} + \frac{C_A}{n_{A0}} + \frac{C_D}{T^2} \left[ \frac{T_A^2}{3} + \left( \frac{n+1}{2} + n_L \right) \frac{T_A^2}{n_{A0}} + \sigma_M^2 \right]. \quad (25)$$

Calculating the first copy of the cost of quality by formula (25) by interval ( $n$ ) and equalizing with zero:

$$\frac{d[c(x)]}{dn} \left\{ \frac{C_C}{n} + \frac{C_A}{n_{A0} \frac{T_A^2}{T_{A0}^2}} + \frac{C_D}{T^2} \left[ \frac{T_A^2}{3} + \left( \frac{n+1}{2} + n_L \right) \frac{T_A^2}{n_{A0} \frac{T_A^2}{T_{A0}^2}} + \sigma_M^2 \right] \right\} = 0, \quad (26)$$

gives the following optimal process check interval:

$$n_{OPT} = \frac{T}{T_{A0}} \sqrt{2n_{A0} \frac{C_C}{C_D}}. \quad (27)$$

Calculating the first cost of quality by formula (25) by adjusting the tolerance ( $T_A$ ) and equating the result to zero:

$$\frac{d[c(x)]}{dT_A}$$



$$\left\{ \frac{C_C}{n} + \frac{C_A}{n_{A0} \frac{T_A^2}{T^2}} + \frac{C_D}{T^2} \left[ \frac{T_A^2}{3} + \left( \frac{n+1}{2} + n_L \right) \frac{T_A^2}{n_{A0} \frac{T_A^2}{T^2}} + \sigma_M^2 \right] \right\} = 0, \quad (28)$$

gives the following optimal tolerance settings:

$$T_{OPT} = \sqrt[4]{T^2 \frac{3C_A T_{A0}^2}{C_D n_{A0}}}. \quad (29)$$

But if these obtained optimal values ( $n_{OPT}$ ) and ( $T_{OPT}$ ) differ significantly from the planned values ( $n_{OPT0}$ ) and ( $T_{OPT0}$ ) then the following average values should be used:

$$n = \frac{n_{A0} + n_{OPT}}{2}, \quad (30)$$

$$T = \frac{T_{A0} + T_{OPT}}{2}. \quad (31)$$

For example, consider the average cost of adjusting the heating temperature parameter when Hot pressing the product. The critical quality characteristic of the quality has an average length ( $\mu_0$ ) and a tolerance of 35  $\mu\text{m}$ , the cost of failure due to exceedance of the tolerance  $c_D = 0.20$  \$ and the number of pressing is  $n_D = 200$  pcs/h, during work of 40 h per week and 48 weeks per year. Checking the process is carried out every 2 hours in the interval  $n = (200 \text{ pcs/h})(2 \text{ h}) = 400$  pcs with control costs  $c_C = 2$  \$ and the number of missed products during the check is  $n_L = 0$  pcs. The process parameter is the heating temperature with the planned tolerance setting  $T_A = \pm 2$  °C, every 2 h, with the adjustment interval  $n_A = (200 \text{ pcs/h})(8 \text{ h}) = 1,600$  pcs and the adjustment costs  $c_A = 2$  \$. The change in temperature for (1 °C) causes the shrinkage dimension to change by 5  $\mu\text{m}/^\circ\text{C}$ , so the temperature parameter  $T = (\pm 35 \mu\text{m})/(5 \mu\text{m}/^\circ\text{C}) = 7$  °C is tolerated.

The planned quality costs according to formula (23) are given, with neglect of control errors ( $\sigma_M^2$ ):

$$\begin{aligned} c(x) &= \frac{C_C}{n} + \frac{C_A}{n_A} + \frac{C_D}{T^2} \left[ \frac{T_A^2}{3} + \left( \frac{n+1}{2} + n_L \right) \frac{T_A^2}{n_A} + \sigma_M^2 \right] = \\ &= \frac{2}{400} + \frac{2}{1,600} + \frac{0.2}{7^2} \left[ \frac{2^2}{3} + \left( \frac{400+1}{2} + 0 \right) \frac{2^2}{1,600} + 0 \right] = \\ &= 0.013730 \text{ $ / kom} \end{aligned}$$

Using the obtained formulas (27), (29), (24) and (25), the following values are calculated: the required optimal process control interval, the optimal size of the process settings, optimal average and total cost of quality:

$$n_{OPT} = \frac{T}{T_{A0}} \sqrt{2n_{A0} \frac{C_C}{c_D}} = \frac{7}{2} \sqrt{2 \cdot 1,600 \frac{2}{0.2}} = 626,$$

$$\begin{aligned} T_{OPT} &= \sqrt[4]{T^2 \frac{3C_A T_{A0}^2}{c_D n_{A0}}} = \sqrt[4]{7^2 \frac{3 \cdot 2 \cdot 2^2}{0.2 \cdot 1,600}} = \\ &= 1.38 \cong 1.5 [^\circ\text{C}]. \end{aligned}$$

Since these obtained optimum values differ significantly from the planned values according to formulas (30) and (31), the following mean values are calculated:

$$n = \frac{n_0 + n_{OPT}}{2} = \frac{400 + 626}{2} = 513 \text{ kom},$$

$$T = \frac{T_0 + T_{OPT}}{2} = \frac{2 + 1.5}{2} = 1.75 [^\circ\text{C}].$$

and the required optimum cost of quality after adjusting:

$$\begin{aligned} c(x) &= \frac{C_C}{n_{OPT}} + \frac{C_A}{n_{A0}} + \\ &+ \frac{C_D}{T^2} \left[ \frac{T_{OPT}^2}{3} + \left( \frac{n_{OPT}+1}{2} + n_L \right) \frac{T_{OPT}^2}{n_{A0}} + \sigma_M^2 \right] = \\ &= \frac{2}{513} + \frac{2}{1,600} + \frac{0.2}{7^2} \left[ \frac{1.75^2}{3} + \left( \frac{513+1}{2} + 0 \right) \frac{1.75^2}{1,600} + 0 \right] = \\ &= 0.011322 \text{ $ / kom}. \end{aligned}$$

So, after adjusting, the savings will be saved with an average cost of  $0.013730 - 0.011322 = 0.002408$  \$/piece or yearly  $(0.002408 \text{ $/pcs})(250 \text{ pcs/h})(40 \text{ h/year}) = 1,155.84$  \$/year.

## CONCLUSION

The demonstrated possible way of calculating optimum cost of quality, similar to the Taguchi method, can enable the reduction of costs in

domestic industrial production. Unfortunately, we do not follow the cost of quality at all, but we only look at the overall balance of the organization. In addition, domestic industrial production is still using an outdated way of organizing work, which is fundamentally different from the way in which jobs are organized in America, especially in terms of tracking quality costs and organizing jobs. We still fill in four workplaces in production: the worker produces the results of the process (semi-product, product, documentation, service), the worker adjusts the production process, the inspector controls the quality of the results of the process, and the worker maintenance service carries out the maintenance of the equipment. In America, all four jobs are filled by only one checker, which performs five jobs: 1. production, 2. quality control, 3. process adjustment, 4. release to accept correct, repair or regrade, and scrap faulty process results, as well as preventive maintenance. In this release we briefly outline possible ways of calculating optimum cost of quality, which can also enable the organization of jobs in a new, American way.

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## OPTIMALNI TROŠKOVI KVALITETA

Pokazani mogući način izračunavanja optimalnih troškova kvaliteta, slično metodi Taguchi, može omogućiti sniženje troškova u domaćoj industrijskoj proizvodnji. Nažalost, mi uopšte ne pratimo troškove kvaliteta već ih jedino sagleđavamo u ukupnom bilansu organizacije. Pored toga domaća industrijska proizvodnja još uvek koristi zastareli način organizovanja rada, koji se bitno razlikuje od načina organizovanja radnih mesta u Americi, naročito u pogledu praćenja troškova kvaliteta i organizovanja radnih mesta. Mi još uvek popunjavamo četiri radna mesta u proizvodnji: radnik proizvodi rezultate procesa (poluproizvod, proizvod, dokumentacija, usluga), regler podešava proizvodni proces, kontrolor kontroliše kvalitet rezultata procesa a služba održavanja vrši održavanje opreme. U Americi, sva ta četiri radna mesta popunjava samo jedan radnik proveravač procesa, koji obavlja pet poslova: proizvodnje, kontrolisanja kvaliteta, podešavanja procesa, puštanja radi prihvatanja ispravnih, popravljanja ili preklasiranja i odbacivanja neispravnih rezultata procesa, uz preventivno održavanje. U ovom saopštenju su ukratko izneti mogući načini izračunavanja optimalnih troškova kvaliteta, koji mogu omogućiti i organizovanje radnih mesta na novi, američki način.

**Ključne reči:** Troškovi kvaliteta, Optimalni troškovi kvaliteta.